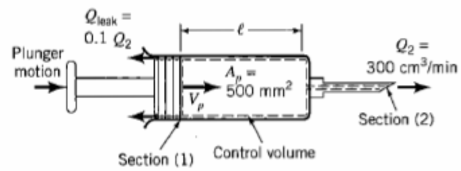
Group HW Problems:

Example 5.8

A syringe (Fig. E5.8) is used to inoculate a cow. The plunger has a face area of 500 mm^2. If the liquid in the syringe is to be injected steadily at a rate of 300 cm^3/min, at what speed should the plunger be advanced? The leakage rate past the plunger is 0.10 times the volume flowrate out of the needle.



Assuming leakage rate is 0.2 times the volume flowrate out of the needle

SOL)

From eq. 5.17

$$\frac{\partial}{\partial t}\int_{CV} \rho d\forall + \dot{m}_2 + \rho Q_{leak} = 0 \qquad (1)$$

First term in eq. (1):

$$\begin{aligned}\int_{CV} \rho d\forall &= \rho\left(\ell A_p + \forall_{needle}\right) \\ &= \rho\left(\ell A_1 + \forall_{needle}\right) \qquad \left(\because A_1 \approx A_p\right)\end{aligned} \qquad (2)$$

Differentiating eq. (2) with respect to time, and recognizing $\rho, A_1, \forall_{needle}$ to be constant over time,

$$\frac{\partial}{\partial t}\int_{CV} \rho d\forall = \rho A_1 \frac{\partial \ell}{\partial t} = -\rho A_1 V_p \qquad \left(\because V_p = -\frac{\partial \ell}{\partial t},\ \ i.e., velocity = \frac{length}{time}\right) \qquad (3)$$

Second term in eq. (1):

$$\dot{m}_2 = \rho Q_2 \qquad (4)$$

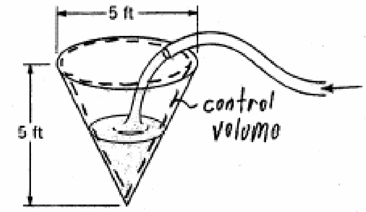
Combining eqs. (1), (3), and (4)

$$-\rho A_1 V_p + \rho Q_2 + \rho Q_{leak} = 0 \qquad (5)$$

$$\therefore\ V_p = \frac{Q_2 + Q_{leak}}{A_1} = \frac{1.2 Q_2}{A_1} \quad \left(\because Q_{leak} = 0.2 Q_2\right) \qquad (6)$$

$$\therefore\ \underline{\underline{V_p = \frac{1.2\left(300\ cm^3/\min\right)}{\left(500\ mm^2\right)}\left(\frac{1000\ mm^3}{cm^3}\right) = 720\ mm/\min}}$$

5.22 Estimate the time required to fill with water a cone-shaped container (see Fig. P5.22) 5 ft high and 5 ft across at the top if the filling rate is 20 gal/min.



■ FIGURE P5.22

From application of the conservation of mass principle to the control volume shown in the figure we have

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \vec{V} \cdot \hat{n} \, dA = 0$$

For incompressible flow

$$\frac{\partial \forall}{\partial t} - Q = 0$$

or

$$\int_0^{\forall} d\forall = Q \int_0^{t} dt$$

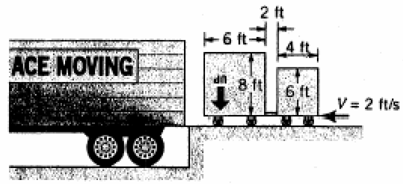
Thus

$$t = \frac{\forall}{Q} = \frac{\pi D^2 h}{12\, Q} = \frac{\pi\,(5\,ft)^2 (5\,ft)\left(1728\,\frac{in.^3}{ft^3}\right)}{(12)\left(20\,\frac{gal}{min}\right)\left(231\,\frac{in.^3}{gal}\right)}$$

and

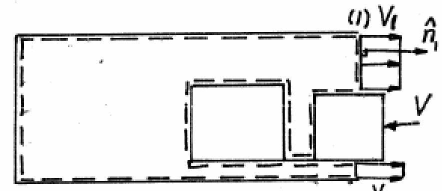
$$t = \underline{\underline{12.2\ min}}$$

5.25 Two 8-ft-wide rectangular crates are wheeled into the trailer portion of a semi-truck at a speed of $V = 2$ ft/s as shown in Fig. P5.25. At time $t = 0$ the front of the first crate is 2 ft from the open back of the trailer. Plot a graph of the air flowrate across the open end of the trailer as a function of time for $0 \le t \le 10$ s.



■ FIGURE P5.25

Consider the deforming control volume shown in the sketch. From Eq.(5.17)



— — deforming control vol.

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \vec{W}\cdot\hat{n}\, dA = 0$$

or since $\rho = \text{constant}$

$$\frac{\partial}{\partial t}\int d\forall + \int \vec{W}\cdot\hat{n}\, dA = 0 \quad \text{or} \quad \frac{d\forall_{cv}}{dt} = -\int \vec{W}\cdot\hat{n}\, dA = -V_1 A_1 = -Q_1 \text{, where}$$

Q_1 = airflow rate across end of trailer.

Also,

$\frac{d\forall}{dt}$ = rate at which the volume of air in the trailer changes with time $= -V A_{crate}$, where A_{crate} = cross-sectional area of the crate that is crossing the end of the trailer.

Thus,

$$-VA_{crate} = -Q_1 \quad \text{or} \quad Q_1 = VA_{crate}\text{, where } V = 2\ ft/s \qquad (1)$$

From the given data

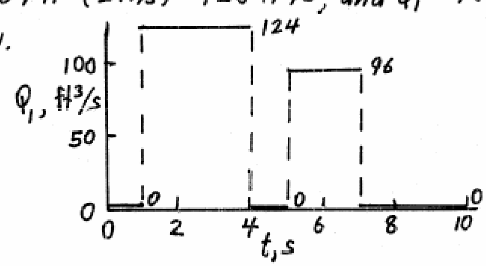
$A_{crate} = 0$ for $0 \le t < 1\,s$; $A_{crate} = (8ft)(8ft) = 64\,ft^2$ for $1s \le t < 4s$;

$A_{crate} = 0$ for $4s \le t < 5s$; $A_{crate} = (6ft)(8ft) = 48\,ft^3$ for $5s \le t \le 7s$

The corresponding flowrates (from Eq. (1)) are

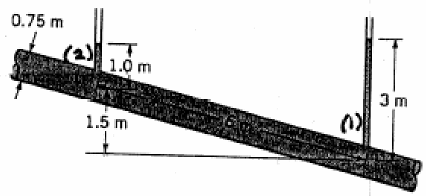
$Q_1 = 0$, $Q_1 = 64\,ft^2(2\,ft/s) = 128\,ft^3/s$, and $Q_1 = 48\,ft^2(2\,ft/s) = 96\,ft^3/s$

as shown below.



5.91

5.91 An incompressible liquid flows steadily along the pipe shown in Fig. P5.91. Determine the direction of flow and the head loss over the 6-m length of pipe.



■ FIGURE P5.91

Assume flow from (1) to (2) and use the energy equation (Eq. 5.84) to get for the contents of the control volume shown:

$$\frac{p_2}{\gamma} + \cancel{\frac{V_2^2}{2g}} + z_2 = \frac{p_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + z_1 + \cancelto{0}{h_s} - h_L$$

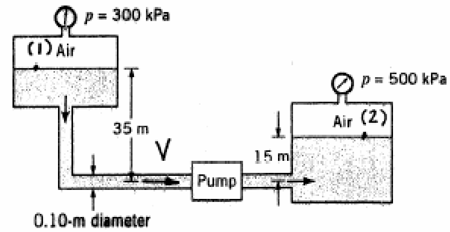
Thus

$$h_L = \frac{p_1}{\gamma} - \frac{p_2}{\gamma} + z_1 - z_2 = 3\,m - 1.0\,m - 1.5\,m = \underline{\underline{0.5\,m}}$$

and since $h_L > 0$, the assumed direction of flow is correct.

The flow is uphill.

5.94 Water is pumped steadily through a 0.10-m-diameter pipe from one closed, pressurized tank to another as shown in Fig. P5.94. The pump adds 4.0 kW to the water and the head loss of the flow is 10 m. Determine the velocity of the water leaving the pipe.



■ FIGURE P5.94

From the energy equation,

(1) $\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$, where $z_1 = 35\,m$, $z_2 = 15\,m$, $V_1 = 0$, $V_2 = 0$, and $h_L = 10\,m$.

Also, $h_s = \frac{\dot{W}_s}{\gamma Q} = \frac{4 \times 10^3 \frac{N \cdot m}{s}}{(9.80 \times 10^3 \frac{N}{m^3})Q} = 0.408/Q$, where $h_s \sim m$ when $Q \sim m^3/s$.

Thus, Eq. (1) becomes

$$\left(\frac{300 \times 10^3 \frac{N}{m^2}}{9.80 \times 10^3 \frac{N}{m^3}}\right) + 35\,m + \left(\frac{0.408}{Q}\,m\right) - 10\,m = \left(\frac{500 \times 10^3 \frac{N}{m^2}}{9.80 \times 10^3 \frac{N}{m^3}}\right) + 15\,m$$

which gives

$Q = 0.0392 \frac{m^3}{s} = AV$

Hence,

$$V = \frac{Q}{A} = \frac{0.0392 \frac{m^3}{s}}{\frac{\pi}{4}(0.1\,m)^2} = \underline{\underline{4.99 \frac{m}{s}}}$$

5.109 A pump is to move water from a lake into a large, pressurized tank as shown in Fig. P5.109 at a rate of 1000 gal in 10 min or less. Will a pump that adds 3 hp to the water work for this purpose? Support your answer with appropriate calculations. Repeat the problem if the tank were pressurized to 3, rather than 2, atmospheres.

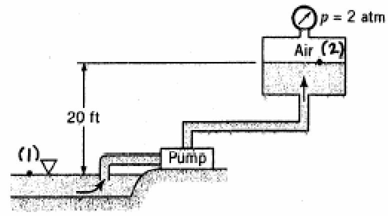


FIGURE P5.109

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_p - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}, \text{ where } p_1 = 0,\ z_1 = 0,\ V_1 = 0, \text{ and } z_2 = 20\,ft.$$

Thus,

(1) $$h_p = h_L + \frac{p_2}{\gamma} + z_2$$

Also,

$$Q = [(1000\,gal)/(10\,min)]\left(\frac{1\,ft^3}{7.48\,gal}\right)\left(\frac{1\,min}{60\,s}\right) = 0.223\,\frac{ft^3}{s}$$

so that

$$h_p = \frac{\dot{W}_p}{\gamma Q} = \frac{(3\,hp)(550\,\frac{ft\cdot lb/s}{hp})}{(62.4\,\frac{lb}{ft^3})(0.223\,\frac{ft^3}{s})} = 119\,ft$$

(a) If $p_2 = 2\,atm = 2(14.7\,\frac{lb}{in.^2})(144\,in.^2/ft^2) = 4{,}230\,\frac{lb}{ft^2}$, then from Eq. (1)

$$h_p = h_L + \frac{4{,}230\,\frac{lb}{ft^2}}{(62.4\,\frac{lb}{ft^3})} + 20\,ft = h_L + 87.8\,ft$$

Thus, if

$h_L \leq h_p - 87.8\,ft = 119\,ft - 87.8\,ft = 31.2\,ft$ the given pump will work for $p_2 = 2\,atm$.

(b) If $p_2 = 3\,atm = 6{,}350\,\frac{lb}{ft^2}$, then

$$h_p = h_L + \frac{6{,}350\,\frac{lb}{ft^3}}{(62.4\,\frac{lb}{ft^3})} + 20\,ft = h_L + 122\,ft$$

Thus, if this pump is to work

$119\,ft = h_L + 122\,ft$, or $h_L \leq -3\,ft$

Since it is not possible to have $h_L < 0$, the pump will not work for $p_2 = 3\,atm$.

5.110

5.110 Water is supplied at 150 ft³/s and 60 psi to a hydraulic turbine through a 3-ft inside diameter inlet pipe as indicated in Fig. P5.110. The turbine discharge pipe has a 4-ft inside diameter. The static pressure at section (2), 10 ft below the turbine inlet, is 10-in. Hg vacuum. If the turbine develops 2500 hp, determine the power lost between sections (1) and (2).

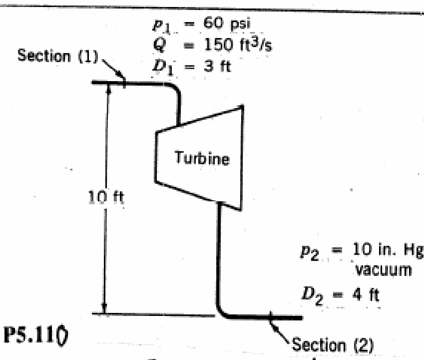


FIGURE P5.110

For flow between sections (1) and (2), Eq. 5.82 leads to

$$\text{power loss} = \rho Q\left[\left(\frac{P_1-P_2}{\rho}\right) + g(z_1-z_2) + \left(\frac{V_1^2-V_2^2}{2}\right)\right] - \dot{W}_{\substack{shaft\\ net\ out}} \qquad (1)$$

From given data

$$P_2 = \frac{(-10\ in.\ Hg)(13.6)\left(1.94\ \frac{slugs}{ft^3}\right)\left(32.2\ \frac{ft}{s^2}\right)}{\left(12\ \frac{in.}{ft}\right)}\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right) = -708\ \frac{lb}{ft^2}$$

Also

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi D_1^2}{4}} = \frac{(4)\left(150\ \frac{ft^3}{s}\right)}{\pi\ (3ft)^2} = 21.22\ \frac{ft}{s}$$

From conservation of mass (Eq. 5.13)

$$V_2 = V_1\frac{A_1}{A_2} = V_1\frac{D_1^2}{D_2^2} = \left(21.22\ \frac{ft}{s}\right)\frac{(3ft)^2}{(4ft)^2} = 11.94\ \frac{ft}{s}$$

From Eq. 1

$$\text{power loss} = \frac{\left(1.94\ \frac{slugs}{ft^3}\right)\left(150\ \frac{ft^3}{s}\right)}{\left(550\ \frac{ft.lb}{s\cdot hp}\right)}\left\{\frac{\left(60\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)+\left(708\ \frac{lb}{ft^2}\right)}{\left(1.94\ \frac{slugs}{ft^3}\right)}\right.$$

$$\left. + \left(32.2\ \frac{ft}{s^2}\right)(10\ ft)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right) + \frac{\left[\left(21.22\ \frac{ft}{s}\right)^2-\left(11.94\ \frac{ft}{s}\right)^2\right]}{2}\left(\frac{1\ lb}{slug.\frac{ft}{s^2}}\right)\right\}$$

$$-2500hp$$

or

$$\text{power loss} = \underline{\underline{301}}\ hp$$

Individual HW Problems:

5.19 As shown in Fig. P5.19, at the entrance to a 3-ft-wide channel the velocity distribution is uniform with a velocity V. Further downstream the velocity profile is given by $u = 4y - 2y^2$, where u is in ft/s and y is in ft. Determine the value of V.

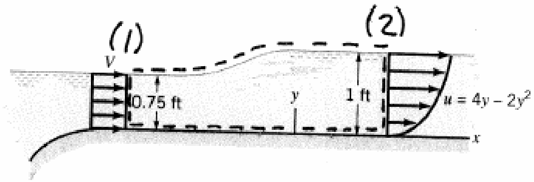


FIGURE P5.19

Use the control volume indicated by the broken lines in the sketch above.

From the conservation of mass principle

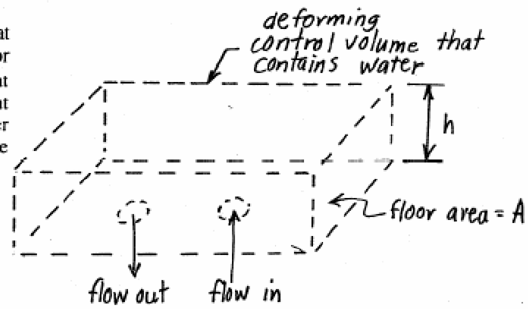
$$Q_1 = Q_2$$

$$V_1 A_1 = \int_{A_2} u\, dA \quad \int_0^{1\,ft} (4y - 2y^2)\, b\, dy$$

$$V(0.75\,ft)\, b = 3\left[\frac{4y^2}{2} - \frac{2y^3}{3}\right]_0^{1\,ft} b = \frac{4b}{3}\ \frac{ft^3}{s}$$

$$V = \frac{4}{3(0.75)} = \underline{\underline{1.78\ \frac{ft}{s}}}$$

5.24 Storm sewer backup causes your basement to flood at the steady rate of 1 in. of depth per hour. The basement floor area is 1500 ft^2. What capacity (gal/min) pump would you rent to (a) keep the water accumulated in your basement at a constant level until the storm sewer is blocked off, (b) reduce the water accumulation in your basement at a rate of 3 in./hr even while the backup problem exists?



For a deforming control volume that contains the water over the basement floor (see sketch above), the conservation of mass principle (Eq. 5.17) leads to

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \vec{V}_r \cdot \hat{n} \, dA = 0$$

or for constant fluid density and area (A)

$$A \frac{dh}{dt} - Q_{in} + Q_{out} = 0 \qquad (1)$$

(a) For part a, Eq. 1 leads to

$$Q_{out} = Q_{in}$$

To evaluate Q_{in}, we use Eq. 1 with $Q_{out} = 0$. Thus,

$$Q_{in} = A\frac{dh}{dt} = (1500\ ft^2)\left(1\ \frac{in.}{hr}\right)\left(\frac{1}{12\ \frac{in.}{ft}}\right) = 125\ \frac{ft^3}{hr}$$

and

$$Q_{out} = \left(125\ \frac{ft^3}{hr}\right)\left(7.48\ \frac{gal}{ft^3}\right)\left(\frac{1}{60\ \frac{min}{hr}}\right) = \underline{\underline{15.6}}\ \frac{gal}{min}$$

(b) For part b, Eq. 1 yields

$$Q_{out} = Q_{in} - A\frac{dh}{dt}$$

$$Q_{out} = 15.6\ \frac{gal}{min} - (1500\ ft^2)\left(-3\ \frac{in.}{hr}\right)\left(\frac{1}{12\ \frac{in.}{ft}}\right)\left(7.48\ \frac{gal}{ft^3}\right)\left(\frac{1}{60\ \frac{min}{hr}}\right)$$

$$Q_{out} = \underline{\underline{62.4}}\ \frac{gal}{min}$$

5.93

5.93 A water siphon having a constant inside diameter of 3 in. is arranged as shown in Fig. P5.93. If the friction loss between A and B is $0.8V^2/2$, where V is the velocity of flow in the siphon, determine the flowrate involved.

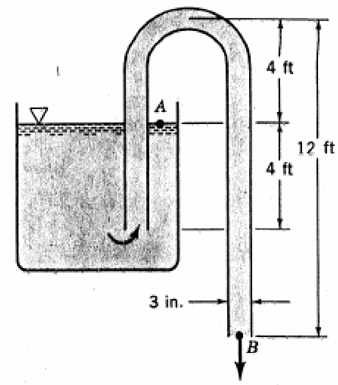


FIGURE P5.93

To determine the flowrate, Q, we use

$$Q = AV = \frac{\pi D^2}{4} V \qquad (1)$$

To obtain V we apply the energy equation (Eq. 5.82) between points A and B in the sketch above. Thus,

$$\cancel{\frac{p_B}{\rho}}^{0} + \frac{V_B^2}{2} + g z_B = \cancel{\frac{p_A}{\rho}}^{0} + \cancel{\frac{V_A^2}{2}}^{0} + g z_A + w_{\substack{shaft\\ net\ in}} - loss$$

or

$$\frac{V^2}{2} + g z_B = g z_A - 0.8\frac{V^2}{2}$$

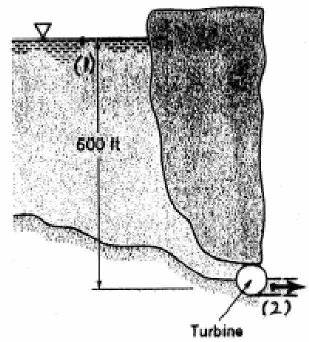
Thus

$$V = \sqrt{\frac{g(z_A - z_B)}{0.9}} = \sqrt{\frac{\left(32.2\ \frac{ft}{s^2}\right)(8\ ft)}{0.9}} = 16.9\ \frac{ft}{s}$$

and with Eq. 1

$$Q = \frac{\pi (3\ in.)^2}{4\left(144\ \frac{in.^2}{ft^2}\right)}\left(16.9\ \frac{ft}{s}\right) = \underline{\underline{0.830\ \frac{ft^3}{s}}}$$

5.108 The hydroelectric turbine shown in Fig. P5.108 passes 8 million gal/min across a head of 600 ft. What is the maximum amount of power output possible? Why will the actual amount be less?



■ FIGURE P5.108

From the energy equation

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where $p_1 = 0$, $p_2 = 0$, and $V_1 = 0$.

Thus,

$$h_s = (z_2 - z_1) + h_L + \frac{V_2^2}{2g}$$

Note: Since this is a turbine, $h_s < 0$. Let $h_T = -h_s$, where $h_T > 0$ and from the above,

$$h_T = (z_1 - z_2) - h_L - \frac{V_2^2}{2g}$$

Also, the power is given by

$$\dot{W}_{turb} = \gamma Q h_T = \gamma Q\left[(z_1 - z_2) - h_L - \frac{V_2^2}{2g}\right]$$

The maximum power would occur if there were no losses ($h_L = 0$) and negligible kinetic energy at the exit ($V_2 \approx 0$; large diameter outlet).

Thus,

$$\dot{W}_{turb\,max} = \gamma Q (z_1 - z_2) = 62.4\frac{lb}{ft^3}\left(8\times10^6\frac{gal}{min}\right)\left(\frac{1\,min}{60s}\right)\left(\frac{1\,ft^3}{7.48\,gal}\right)(600\,ft)$$

$$= 6.67\times10^8\,\frac{ft\cdot lb}{s}\left(\frac{1\,hp}{550\,\frac{ft\cdot lb}{s}}\right) = \underline{\underline{1.21\times10^6\,hp}}$$

5.111 Gasoline ($SG = 0.68$) flows through a pump at 0.12 m³/s as indicated in Fig. P5.111. The loss between sections (1) and (2) is loss $= h_L g = 0.3\, V_1^2/2$. What will the difference in pressures between sections (1) and (2) be if 20 kW is delivered by the pump to the fluid?

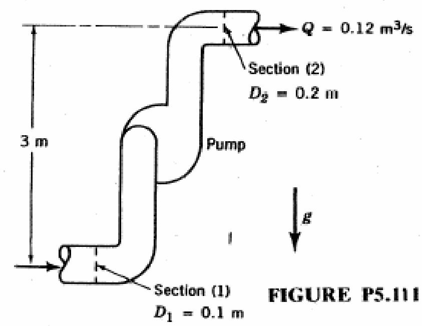


FIGURE P5.111

From Eq. 5.82 we get for the flow from section (1) to section (2)

$$p_1 - p_2 = \rho\left[\frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) - w_{\substack{shaft\\ net\ in}} + loss\right] \qquad (1)$$

From the volume flowrate we obtain

$$V_2 = \frac{Q}{A_2} = \frac{Q}{\frac{\pi D_2^2}{4}} = \frac{(0.12\ \frac{m^3}{s})^2}{\frac{\pi\ (0.2\,m)^2}{4}} = 3.82\ \frac{m}{s}$$

and from conservation of mass (Eq. 5.13) it follows that

$$V_1 = V_2\ \frac{A_2}{A_1} = V_2\ \frac{D_2^2}{D_1^2} = \left(3.82\ \frac{m}{s}\right)\frac{(0.2\,m)^2}{(0.1\,m)^2} = 15.28\ \frac{m}{s}$$

Also

$$w_{\substack{shaft\\ net\ in}} = \frac{\dot{W}_{\substack{shaft\\ net\ in}}}{\rho Q} = \frac{\left(20{,}000\ \frac{N\cdot m}{s}\right)}{(0.68)(999\ \frac{kg}{m^3})(0.12\ \frac{m^3}{s})} = 245.3\ \frac{N\cdot m}{kg}$$

and

$$loss = 0.3\ \frac{V_1^2}{2} = \frac{(0.3)\left(15.28\ \frac{m}{s}\right)^2}{2}\left(1\ \frac{N}{kg\cdot\frac{m}{s^2}}\right) = 35.02\ \frac{N\cdot m}{kg}$$

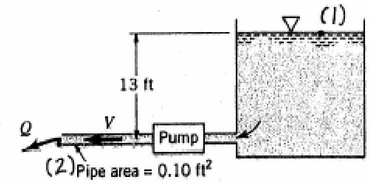
From Eq. 1 then

$$p_1 - p_2 = (0.68)(999\ \frac{kg}{m^3})\left\{\left[\frac{(3.82\ \frac{m}{s})^2 - (15.28\ \frac{m}{s})^2}{2} + (9.81\ \frac{m}{s^2})(3\,m)\right]\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right) - 245.3\ \frac{N\cdot m}{kg} + 35.02\ \frac{N\cdot m}{kg}\right\}$$

or

$$p_1 - p_2 = -197{,}000\ \frac{N}{m^2} = -197\ kPa$$

5.115 Water is pumped from the large tank shown in Fig. P5.115. The head loss is known to be equal to $4V^2/2g$ and the pump head is $h_p = 20 - 4Q^2$, where h_p is in ft when Q is in ft^3/s. Determine the flowrate.



■ FIGURE P5.115

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_p - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g},$$ where $p_1 = p_2 = 0$, $z_1 = 13\,ft$, $z_2 = 0$, and $V_1 = 0$.

Thus,

(1) $$z_1 + h_p - h_L = \frac{V_2^2}{2g}$$

Also,

$$h_L = 4\frac{V^2}{2g} = 4\frac{V_2^2}{2g} = 4\frac{(Q/A_2)^2}{2g}$$ since $V_2 = \frac{Q}{A_2}$

Hence, Eq. (1) becomes

$$z_1 + (20 - 4Q^2) - 4\frac{(Q/A_2)^2}{2g} = \frac{(Q/A_2)^2}{2g}$$

or

$$\left[\left(\frac{5}{2g A_2^2}\right) + 4\right] Q^2 = 20 + z_1$$, where $g \sim \frac{ft}{s^2}$, $A_2 \sim ft^2$, and $Q \sim \frac{ft^3}{s}$

Thus, with the given data

$$\left[\left(\frac{5}{2(32.2\frac{ft}{s^2})(0.1\,ft^2)^2}\right) + 4\right] Q^2 = 20 + 13\,ft$$

or

$$\underline{\underline{Q = 1.67\,\frac{ft^3}{s}}}$$